

CHEN
460

November 24, 2014

Computer Project

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Fall 2014

D) At steady-state

$$A: \bar{Q} = -C_{12} \operatorname{sgn}(\bar{h}_1 - \bar{h}_2) \sqrt{|\bar{h}_1 - \bar{h}_2|} + \bar{Q}_{in}$$

$$B: \bar{Q} = C_{12} \operatorname{sgn}(\bar{h}_1 - \bar{h}_2) \sqrt{|\bar{h}_1 - \bar{h}_2|} - C_{23} \operatorname{sgn}(\bar{h}_2 - \bar{h}_3) \sqrt{|\bar{h}_2 - \bar{h}_3|}$$

$$C: \bar{Q} = C_{23} \operatorname{sgn}(\bar{h}_2 - \bar{h}_3) \sqrt{|\bar{h}_2 - \bar{h}_3|} - C_3 \sqrt{\bar{h}_3}$$

A+B

$$\bar{Q}_{in} = C_{23} \underbrace{\operatorname{sgn}(\bar{h}_2 - \bar{h}_3)}_{x = \bar{h}_2 - \bar{h}_3 > 0} \sqrt{|\bar{h}_2 - \bar{h}_3|} \quad @ ss \bar{h}_1 > \bar{h}_2 > \bar{h}_3$$
$$\operatorname{sgn}(x) = 1$$

$$\bar{Q}_{in} = C_{23} \sqrt{\bar{h}_2 - \bar{h}_3} \quad \left(\frac{\bar{Q}_{in}}{C_{23}} \right)^2 = \bar{h}_2 - \bar{h}_3 \quad (D)$$

B+C

$$C_3 \sqrt{\bar{h}_3} = C_{12} \underbrace{\operatorname{sgn}(\bar{h}_1 - \bar{h}_2)}_{x = \bar{h}_1 - \bar{h}_2 > 0} \sqrt{|\bar{h}_1 - \bar{h}_2|} \quad @ ss \bar{h}_1 > \bar{h}_2$$
$$\operatorname{sgn}(x) = 1$$

$$(C_3 \sqrt{\bar{h}_3})^2 = (C_{12} \sqrt{\bar{h}_1 - \bar{h}_2})^2$$

$$(E) C_3^2 \bar{h}_3 = C_{12}^2 (\bar{h}_1 - \bar{h}_2) = C_{12}^2 \bar{h}_1 - C_{12}^2 \bar{h}_2$$

$$A: \bar{Q}_{in} = C_{12} \operatorname{sgn}(\bar{h}_1 - \bar{h}_2) \sqrt{\bar{h}_1 - \bar{h}_2} \quad @ ss \bar{h}_1 > \bar{h}_2$$
$$x = \bar{h}_1 - \bar{h}_2 > 0$$
$$\operatorname{sgn}(x) = 1$$

$$(\bar{Q}_{in})^2 = (C_{12} \sqrt{\bar{h}_1 - \bar{h}_2})^2$$

$$(F) \bar{Q}_{in}^2 = C_{12}^2 (\bar{h}_1 - \bar{h}_2)$$

From E & E

$$\bar{Q}_{in}^2 = C_3^2 \bar{h}_3$$

$$\boxed{\bar{h}_3 = \frac{\bar{Q}_{in}^2}{C_3^2}}$$

D & \bar{h}_3

$$\bar{h}_2 = \frac{\bar{Q}_{in}^2}{C_{23}^2} + \bar{h}_3 = \frac{\bar{Q}_{in}^2}{C_{23}^2} + \frac{\bar{Q}_{in}^2}{C_3^2} = \left(\frac{1}{C_{23}^2} + \frac{1}{C_3^2} \right) \bar{Q}_{in}^2$$

$$\boxed{\bar{h}_2 = \bar{Q}_{in}^2 \left(\frac{1}{C_{23}^2} + \frac{1}{C_3^2} \right)}$$

E & \bar{h}_2 & \bar{h}_3

$$C_3^2 \bar{h}_3 + C_{12}^2 \bar{h}_2 = C_{12}^2 \bar{h}_1$$

$$C_3^2 \frac{\bar{Q}_{in}^2}{C_3^2} + C_{12}^2 \bar{Q}_{in}^2 \left(\frac{1}{C_{23}^2} + \frac{1}{C_3^2} \right) = C_{12}^2 \bar{h}_1 = \bar{Q}_{in}^2 \left(1 + \frac{C_{12}^2}{C_{23}^2} + \frac{C_{12}^2}{C_3^2} \right)$$

$$\boxed{\bar{h}_1 = \frac{\bar{Q}_{in}^2}{C_{12}^2} + \frac{\bar{Q}_{in}^2}{C_{23}^2} + \frac{\bar{Q}_{in}^2}{C_3^2}}$$

$$\bar{h}_2 = 23.11 \text{ cm}$$

$$\bar{h}_3 = 5.99 \text{ cm}$$

$$\bar{h}_1 = 94 \text{ cm}$$

PART 3

$$\textcircled{2} \quad \frac{\partial}{\partial x} (\operatorname{sgn}(x) \sqrt{|x|}) = \frac{1}{2|x|} \quad \text{at Eq 5.7.1}$$

$$\begin{aligned} & \operatorname{sgn}(h_1 - h_2) \sqrt{|h_1 - h_2|} \approx \sqrt{|h_1 - h_2|} \\ & \approx \sqrt{|h_1 - \bar{h}_2|} + \hat{h}_1 \left(\frac{1}{2\sqrt{|h_1 - \bar{h}_2|}} \right) - \hat{h}_2 \left(\frac{1}{2\sqrt{|h_1 - \bar{h}_2|}} \right) \end{aligned}$$

$\hat{h}_1 = h_1 - \bar{h}_1$
 $\hat{h}_2 = h_2 - \bar{h}_2$
 $\hat{h}_3 = h_3 - \bar{h}_3$

$$\begin{aligned} & \operatorname{sgn}(h_2 - h_3) \sqrt{|h_2 - h_3|} \\ & \approx \sqrt{|h_2 - \bar{h}_3|} + \hat{h}_2 \left(\frac{1}{2\sqrt{|h_2 - \bar{h}_3|}} \right) - \hat{h}_3 \left(\frac{1}{2\sqrt{|h_2 - \bar{h}_3|}} \right) \end{aligned}$$

t_L

$$\frac{dh_1}{dt} = -C_{12} \left[\sqrt{|h_1 - \bar{h}_2|} + (h_1 - \bar{h}_1) \left(\frac{1}{2\sqrt{|h_1 - \bar{h}_2|}} \right) - (h_2 - \bar{h}_2) \left(\frac{1}{2\sqrt{|h_1 - \bar{h}_2|}} \right) \right] + 0$$

From Eq F $\bar{Q}_{in} = C_{12} \sqrt{|h_1 - \bar{h}_2|} \quad -\bar{Q}_{in} = -C_{12} \sqrt{|h_1 - \bar{h}_2|}$

$$\frac{dh_1}{dt} = -\frac{C_{12}}{2A\sqrt{|h_1 - \bar{h}_2|}} (h_1 - \bar{h}_1) + \frac{C_{12}}{2A\sqrt{|h_1 - \bar{h}_2|}} (h_2 - \bar{h}_2) + \frac{(\bar{Q}_{in} - -\bar{Q}_{in})}{A}$$

2

$$\frac{dh_2}{dt} = \underbrace{C_{12} \sqrt{|h_1 - \bar{h}_2|}}_{\leftarrow \bar{Q}_{in}} + \frac{C_{12} (h_1 - \bar{h}_1)}{2\sqrt{|h_1 - \bar{h}_2|}} - \frac{C_{12} (h_2 - \bar{h}_2)}{2\sqrt{|h_1 - \bar{h}_2|}} - \underbrace{C_{23} \sqrt{|h_2 - \bar{h}_3|}}_{\bar{Q}_{in} \text{ term } \cancel{\text{out}} \text{ (D)}} - \frac{(h_2 - \bar{h}_2) C_{23}}{2\sqrt{|h_2 - \bar{h}_3|}} + \frac{C_{23} (h_3 - \bar{h}_3)}{2\sqrt{|h_2 - \bar{h}_3|}}$$

\bar{Q}_{in} cancels out

$$A \frac{dh_2}{dt} = \frac{C_{12}}{2\sqrt{|h_1 - \bar{h}_2|}} (h_1 - \bar{h}_1) - \frac{C_{12}}{2\sqrt{|h_1 - \bar{h}_2|}} (h_2 - \bar{h}_2) - \frac{C_{23}}{2\sqrt{|h_2 - \bar{h}_3|}} (h_2 - \bar{h}_2) + \frac{C_{23} (h_3 - \bar{h}_3)}{2\sqrt{|h_2 - \bar{h}_3|}}$$

$$\sqrt{h_3} \approx \sqrt{\bar{h}_3} + \frac{1}{2\sqrt{\bar{h}_3}} (h_3 - \bar{h}_3)$$

$$Q_{out} - \bar{Q}_{out} = \hat{Q}_{out}$$

$$Q_{out} - \bar{Q}_{out} = \frac{C_3}{2\sqrt{\bar{h}_3}} (h_3 - \bar{h}_3)$$

$$Q_{out} = C_3 \sqrt{\bar{h}_3} + \frac{C_3}{2\sqrt{\bar{h}_3}} (h_3 - \bar{h}_3) \quad \bar{Q}_{out} = C_3 \sqrt{\bar{h}_3}$$

(3)

3) C

$$\frac{d\bar{h}_3}{dt} = \underbrace{C_{23}\sqrt{\bar{h}_2 - \bar{h}_3}}_{EQ D} + \frac{C_{23}(\bar{h}_2 - \bar{h}_3)}{2\sqrt{\bar{h}_2 - \bar{h}_3}} - \frac{C_{23}(\bar{h}_3 - \bar{h}_3)}{2\sqrt{\bar{h}_2 - \bar{h}_3}} - C_3\sqrt{\bar{h}_3} - \frac{C_3(\bar{h}_3 - \bar{h}_3)}{2\sqrt{\bar{h}_3}}$$

\hat{Q}_{in} cancel out $\sqrt{\bar{h}_3} = \frac{\hat{Q}_{in}}{C_3}$

$$A\frac{d\bar{h}_3}{dt} = \frac{C_{23}}{2\sqrt{\bar{h}_2 - \bar{h}_3}} (\bar{h}_2 - \bar{h}_3) - \frac{C_{23}}{2\sqrt{\bar{h}_2 - \bar{h}_3}} (\bar{h}_3 - \bar{h}_3) - \frac{C_3}{2\sqrt{\bar{h}_3}} (\bar{h}_3 - \bar{h}_3)$$

in deviation variable form

$$A\frac{d\hat{h}_1}{dt} = \frac{-C_{12}}{2\sqrt{\bar{h}_1 - \bar{h}_2}} (\hat{h}_1 - \hat{h}_2) + \hat{Q}_{in}$$

$$A\frac{d\hat{h}_2}{dt} = \frac{C_{12}}{2\sqrt{\bar{h}_1 - \bar{h}_2}} (\hat{h}_1 - \hat{h}_2) - \frac{C_{23}}{2\sqrt{\bar{h}_2 - \bar{h}_3}} (\hat{h}_2 - \hat{h}_3) \quad \& \hat{Q}_{out} = \frac{C_3 \bar{h}_3}{2\sqrt{\bar{h}_3}}$$

$$A\frac{d\hat{h}_3}{dt} = \frac{C_{23}}{2\sqrt{\bar{h}_2 - \bar{h}_3}} (\hat{h}_2 - \hat{h}_3) - \frac{C_3}{2\sqrt{\bar{h}_3}} \hat{h}_3$$

$$k_1 = \frac{C_{12}}{2\sqrt{\bar{h}_1 - \bar{h}_2}} \quad k_2 = \frac{C_{23}}{2\sqrt{\bar{h}_2 - \bar{h}_3}} \quad k_3 = \frac{C_3}{2\sqrt{\bar{h}_3}}$$

$$A\frac{d\hat{h}_1}{dt} = -k_1(\hat{h}_1 - \hat{h}_2) + \hat{Q}_{in}$$

$$A\frac{d\hat{h}_2}{dt} = k_1(\hat{h}_1 - \hat{h}_2) - k_2(\hat{h}_2 - \hat{h}_3)$$

$$A\frac{d\hat{h}_3}{dt} = k_2(\hat{h}_2 - \hat{h}_3) - k_3 \hat{h}_3$$

$$\hat{Q}_{out} = k_3 \hat{h}_3$$

$$③ G_p(s) = G_1 G_2 G_3 \text{ (series)}$$

input \hat{Q}_{in}
output \hat{Q}_{out}

$$\frac{d}{dt} \begin{bmatrix} \hat{h}_1 \\ \hat{h}_2 \\ \hat{h}_3 \end{bmatrix} = \begin{bmatrix} -k_1/A & k_1/A & 0 \\ k_1/A & -k_1-k_2/A & k_2/A \\ 0 & k_2/A & -k_2-k_3/A \end{bmatrix} \begin{bmatrix} \hat{h}_1 \\ \hat{h}_2 \\ \hat{h}_3 \end{bmatrix} + \begin{bmatrix} \frac{1}{A} \\ 0 \\ 0 \end{bmatrix} \hat{Q}_{in}$$

$$\hat{Q}_{out} = [0 \ 0 \ k_3] \begin{bmatrix} \hat{h}_1 \\ \hat{h}_2 \\ \hat{h}_3 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \hat{Q}_{in}$$

$$\bar{Q}_{in}^2 \left[\frac{1}{C_{12}^2} + \frac{1}{C_{23}^2} + \frac{1}{C_3^2} \right] = \bar{h}_1$$

$$\bar{Q}_{in} = \left(\frac{\bar{h}_1}{\frac{1}{C_{12}^2} + \frac{1}{C_{23}^2} + \frac{1}{C_3^2}} \right)^{1/2}$$

$$\hat{h}_n = [1 \ 0 \ 0] \begin{bmatrix} \hat{h}_1 \\ \hat{h}_2 \\ \hat{h}_3 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \hat{Q}_{in}$$

$$G_1 = \frac{0.006499s^2 + 0.03343s + 3.176 \times 10^{-6}}{s^3 + 0.05266s^2 + 0.00807s + 1.66 \times 10^{-6}}$$

poles are equal to $-1/\tau_1, -1/\tau_2, -1/\tau_3$

zeros are equal to $-1/\theta_1, -1/\theta_2$

$$K_p = G_p @ s=0 \quad G_p(0) = K_p(1)$$

$$G_1(0) = \frac{3.17 \times 10^{-6}}{1.66 \times 10^{-6}} = 1.906 = K_p$$

) From MATLAB zeros = -0.0389 & -0.0126
poles = -0.039146, -0.0170, -0.0025

$$\begin{aligned}\tau_1 &= +25.5621 \quad \tau_2 = 58.6838 \quad \tau_3 = 400.0931 && \text{Poles} \\ \theta_1 &= +25.6943 \quad \theta_2 = 79.5725 && \text{zeros}\end{aligned}$$

$$G_p(s) = 1.906 \frac{(25.7s+1)(79.6s+1)}{(25.6s+1)(58.7s+1)(400.1s+1)}$$

- All poles are negative and real which means the system is stable. Real negative poles indicate exponential decay.
- The zeros are negative (left-half plane zeros), which indicates there is a possibility of overshoot.

```

%% Parameters
a=154; %% cm2
C12=10.1; %%cm^(5/2)/s
C23=11; %%cm^(5/2)/s
C3=19.7; %%cm^(5/2)/s

%% Steady State Parameters Part 1
h1s=44; %%cm
Qins=(h1s/(1/(C12^2)+1/(C23^2)+1/(C3^2)))^(1/2);
h2s=Qins^2*(1/(C23^2)+1/(C3^2));
h3s=(Qins/C3)^2

%%Constants
k1=C12/(2*sqrt(h1s-h2s));
k2=C23/(2*sqrt(h2s-h3s));
k3=C3/(2*sqrt(h3s));

%%Matrices Part 3
A=[-1*k1/a k1/a 0;k1/a -(k1+k2)/a k2/a -(k2+k3)/a];
B=[1/a;0;0];
C=[1 0 0];
D=[0];

sys=ss(A,B,C,D);

%%Part 3
G1=tf(sys)

P=pole(G1)

Z=zero(G1)

%%Tau values
tau1=-1/P(1)
tau2=-1/P(2)
tau3=-1/P(3)

%%Theta values
thetal=-1/Z(1)
theta2=-1/Z(2)

%%Process Gain
Kp=dcgain(sys)

%%Part 4 Step increase in feed flow rate
Qin=5+Qins; %%cm^3/s (part 4)

%%Nonlinear
[T,h]=ode15s(@(t,h)ProjectModel4(h,C12,C23,C3,Qin,a,t),[0 100],[h1s h2s h3s]);

```

```
%%Linear
[T,L]=ode15s(@(t,L)LinearModel4(L,t,C12,C23,C3,a,h1s,h2s,h3s,Qin,Qins),[0 100],[h1s h2s h3s]);
%%Plotting

plot(T,h(:,1),'b',T,L(:,1),'m')
%ylim([20,45])
%xlim([0 100])
hold on
a=legend('level h1','levelL1',4);
set(a,'Interpreter','none')
title('step response of three interacting tanks: h1 Nonlinear vs. linear')
ylabel('tank level (ft)')
xlabel('time (sec)')

figure
plot(T,h(:,2),'b',T,L(:,2),'m')
%ylim([14,24])
%xlim([0 100])
hold on
a=legend('level h2','levelL2',4);
set(a,'Interpreter','none')
title('step response of three interacting tanks: h2 Nonlinear vs. linear')
ylabel('tank level (ft)')
xlabel('time (sec)')

figure
plot(T,h(:,3),'b',T,L(:,3),'m')
%ylim([4,5.8])
%xlim([0 100])
hold on
a=legend('level h3','levelL3',4);
set(a,'Interpreter','none')
title('step response of three interacting tanks: h3 Nonlinear vs. linear')
ylabel('tank level (ft)')
xlabel('time (sec)')

hold on
```

`h2s =`
23.1064

`h3s =`
5.4919

Transfer function:
$$\frac{0.006494 s^2 + 0.0003343 s + 3.176e-006}{s^3 + 0.05866 s^2 + 0.000807 s + 1.666e-006}$$

`P =`
-0.0391
-0.0170
-0.0025

`Z =`
-0.0389
-0.0126

`tau1 =`
25.5621

`tau2 =`
58.6838

`tau3 =`
400.0931

`theta1 =`
25.6943

`theta2 =`

79.5725

Kp =

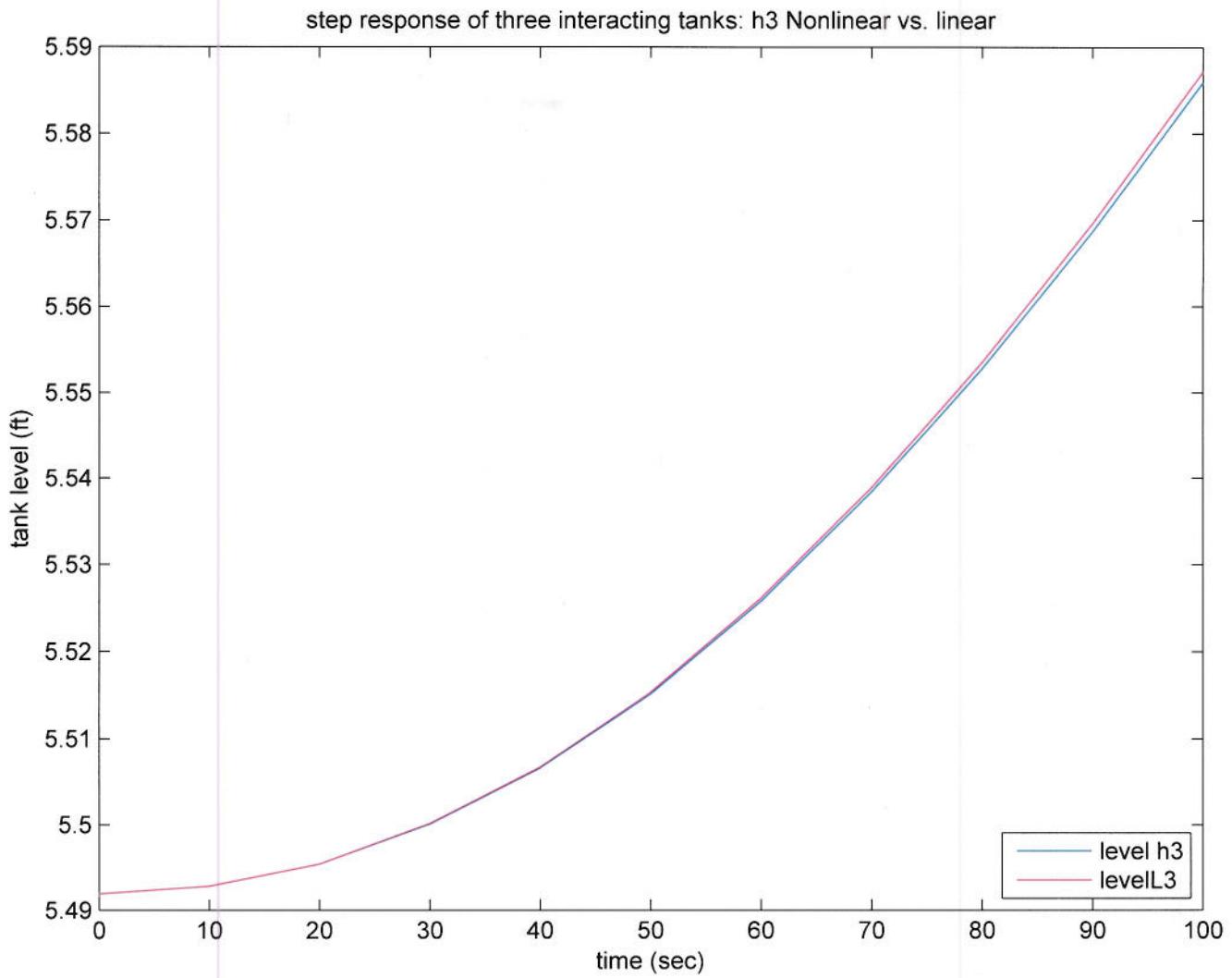
1.9061

>>

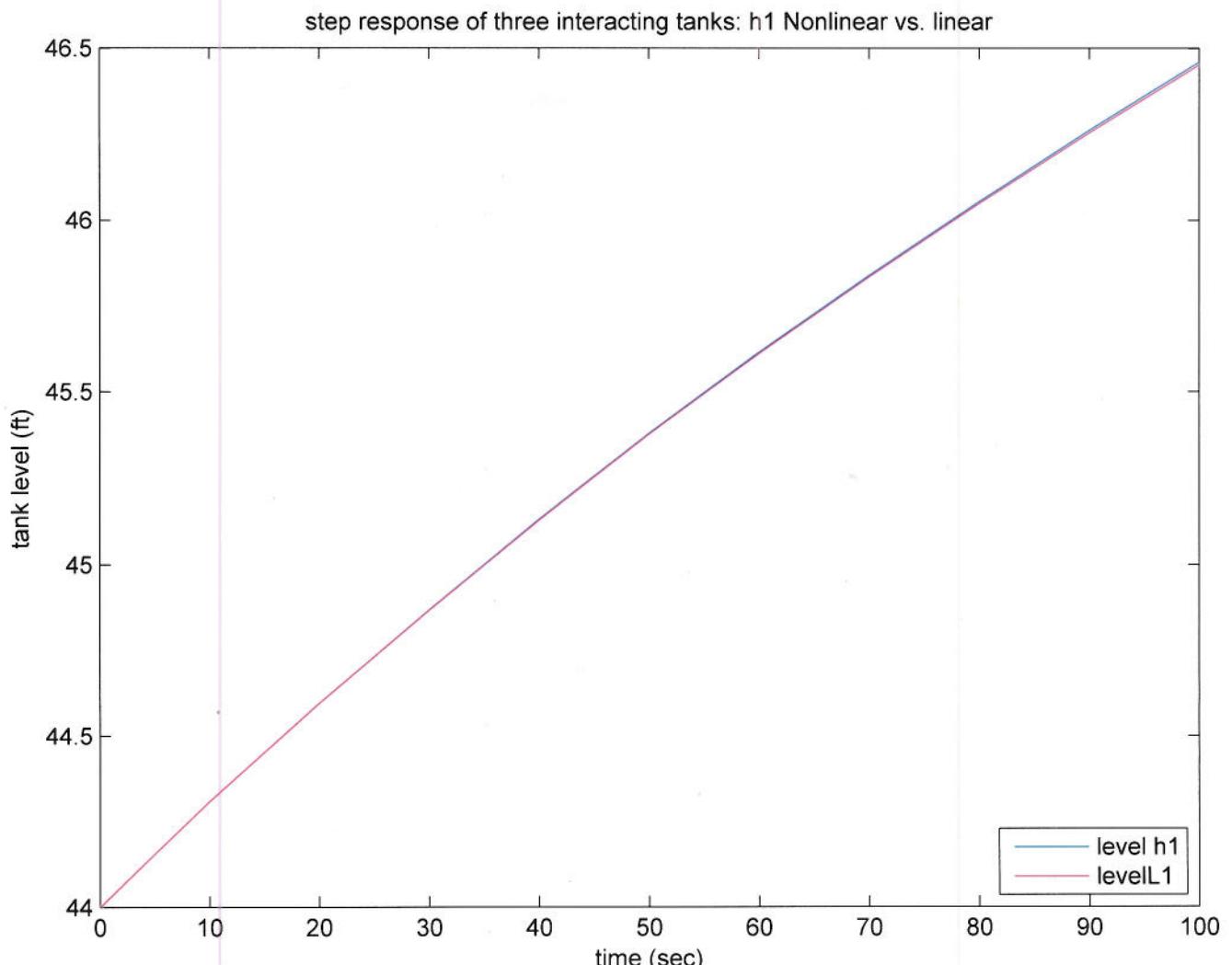
```
function dLdt = LinearModel4(L,t,C12,C23,C3,a,h1s,h2s,h3s,Qin,Qins)
dLdt(1)=-C12/(2*a*sqrt(h1s-h2s))*(L(1)-h1s)+C12/(2*a*sqrt(h1s-h2s))*(L(2)-h2s)+(Qin-Qins)/a;
dLdt(2)= C12/(2*a*sqrt(h1s-h2s))*(L(1)-h1s)-C12/(2*a*sqrt(h1s-h2s))*(L(2)-h2s)-C23/(2*a*sqrt(h2s-h3s))*(L(2)-h2s)+C23/(2*a*sqrt(h2s-h3s))*(L(3)-h3s);
dLdt(3)= C23/(2*a*sqrt(h2s-h3s))*(L(2)-h2s)-C23/(2*a*sqrt(h2s-h3s))*(L(3)-h3s)-C3/(2*a*sqrt(h3s))*(L(3)-h3s);
dLdt=dLdt';
```

```
%%% Part 4
function dhdt = ProjectModel4(h,C12,C23,C3,Qin,a,t)
dhdt(1)= (-C12*sign(h(1)-h(2))*sqrt(abs(h(1)-h(2)))+Qin)/a;
dhdt(2)= (C12*sign(h(1)-h(2))*sqrt(abs(h(1)-h(2)))-C23*sign(h(2)-h(3))*sqrt(abs(h(2)-h(3))))/a;
dhdt(3)=(C23*sign(h(2)-h(3))*sqrt(abs(h(2)-h(3)))-C3*sqrt(h(3)))/a;
dhdt=dhdt';
```

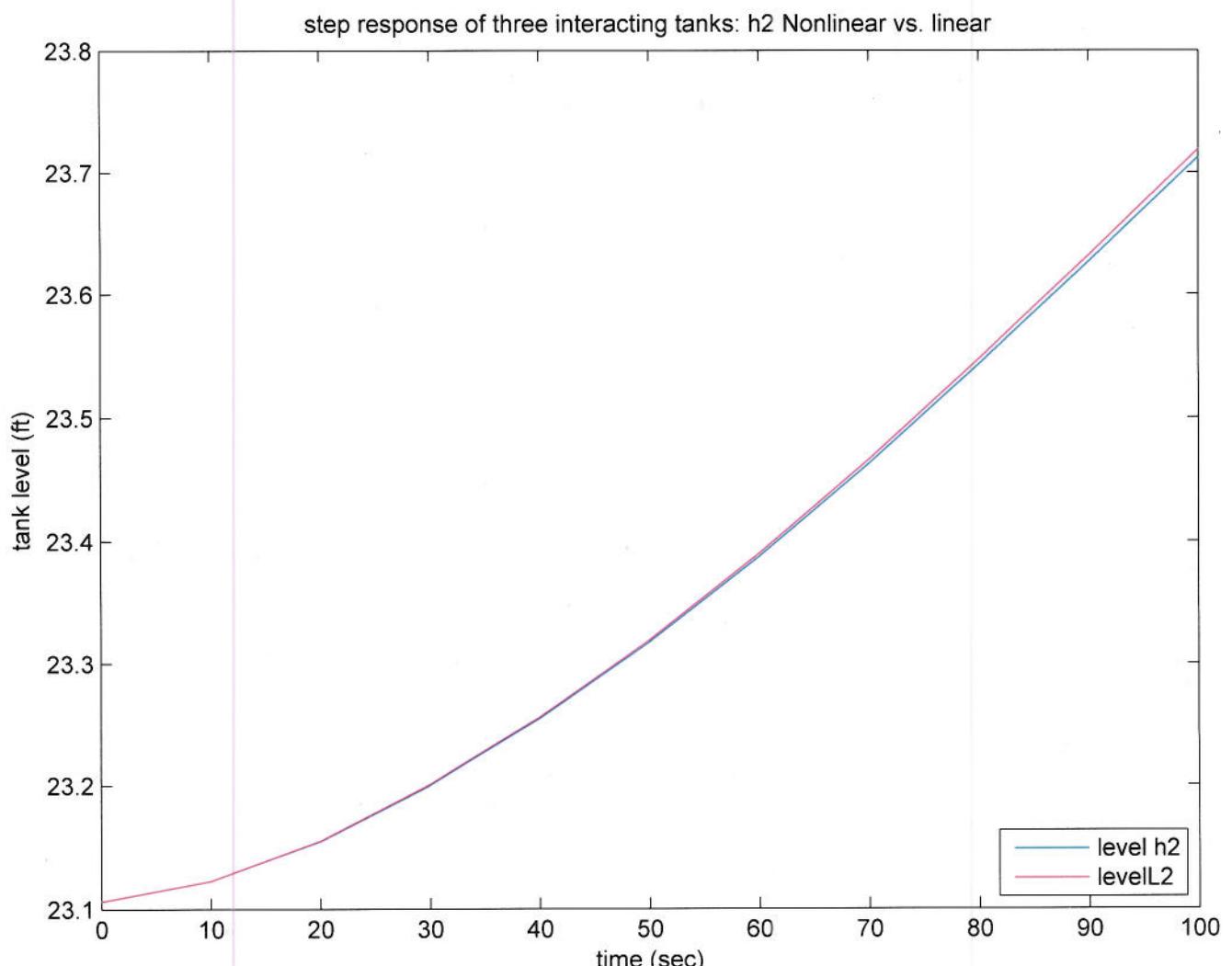
PART 4



PART 4



PART 4



④ nonlinear: "Project Model 9"

For linear model $h_1 = l_1 \quad h_2 = l_2 \quad h_3 = l_3$

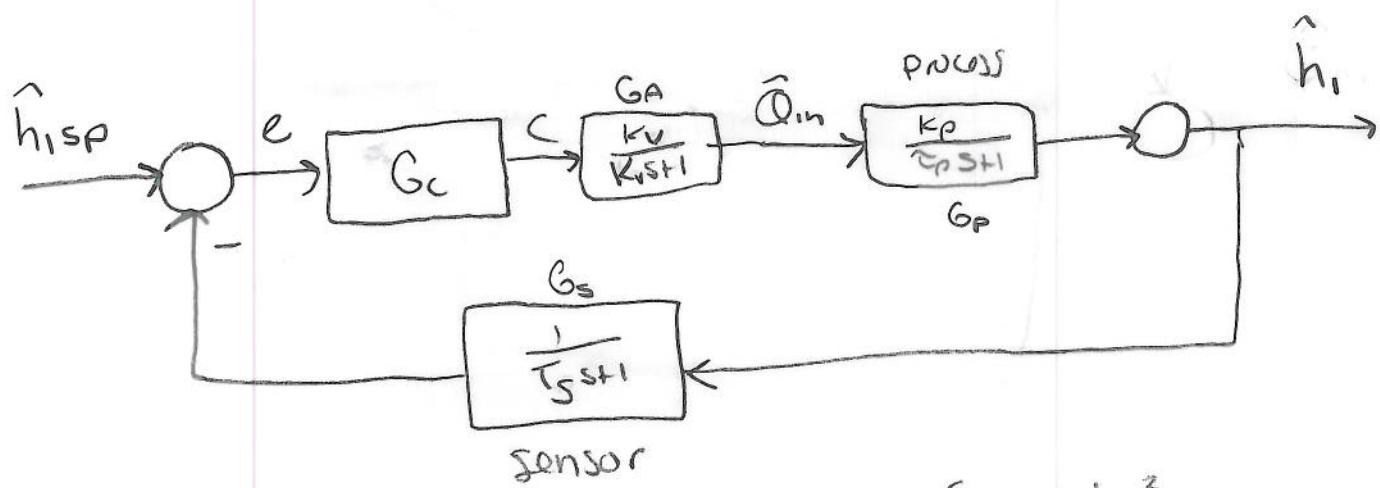
"Linear model"

• Linear model predicts the real model well.

⑤ $h_{1S} = 44 \text{ cm}$ New set point: $h_{1NS} = 52 \text{ cm}$

$$\text{Step change} = 52 - 44 = 8 \text{ cm} = M \quad U(s) = \frac{8}{s}$$

$$\hat{h}_1(s) = \hat{h}_{1SP}(s) G_{CL}(s)$$



P-only $G_c = K_c$

$$G_{CL} = \frac{G_c G_A G_p}{G_c G_a G_p G_s + 1} \quad \text{from part 3}$$

$$G_{CL} = \frac{G_A G_p K_c}{G_p G_A G_s K_c + 1} = \frac{G_p K_c}{G_p K_c + 1}$$

Assume $G_A = G_s = 1$
b/c they are fast

$$\hat{h}_1(s) = \frac{G_p K_c}{G_p K_c + 1} \hat{h}_{1SP}(s)$$

$$G_{CL} = \frac{(0_1 s + 1)(0_2 s + 1) | K_p K_c |}{(T_1 s + 1)(T_2 s + 1)(T_3 s + 1)} \\ \frac{1 + \frac{K_p (0_1 s + 1)(0_2 s + 1) K_c}{(T_1 s + 1)(T_2 s + 1)(T_3 s + 1)}}{1}$$

$$G_{CL} = \frac{(\theta_1 s + 1)(\theta_2 s + 1) K_p K_c}{(\tau_1 s + 1)(\tau_2 s + 1)(\tau_3 s + 1) + K_p K_c (\theta_1 s + 1)(\theta_2 s + 1)} = \frac{\hat{h}_1(s)}{h_{ISP}(s)}$$

Simplified

$$G_{CL} = \frac{(\theta_2 s + 1) K_p K_c}{(\tau_2 s + 1)(\tau_3 s + 1) + K_p K_c (\theta_2 s + 1)}$$

Step change : $\hat{h}_{ISP} = \frac{A}{5}$ $A = 8 \text{ cm}$

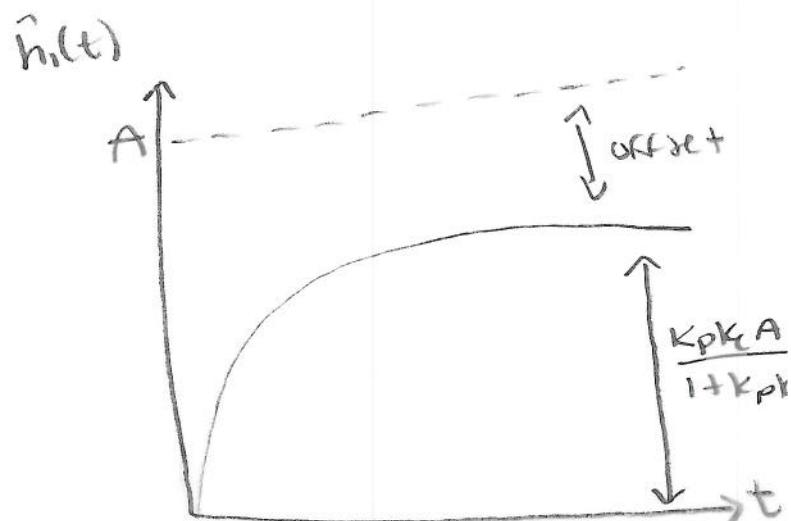
$$\hat{h}_1(s) = G_{CL}(s) \times \hat{h}_{ISP}(s)$$

Final Value Theorem :

$$\lim_{t \rightarrow \infty} \hat{h}_1(t) = \lim_{s \rightarrow 0} [s \hat{h}_1(s)] = \lim_{s \rightarrow 0} G_{CL} \cdot A$$

$$= \lim_{s \rightarrow 0} \frac{(\theta_2 s + 1) K_p K_c \cdot A}{(\tau_2 s + 1)(\tau_3 s + 1) + K_p K_c (\theta_2 s + 1)}$$

$$\lim_{t \rightarrow \infty} \hat{h}_1(t) = \frac{K_p K_c A}{1 + K_p K_c}$$



$$\text{Offset} = A - \frac{K_p K_c A}{1 + K_p K_c}$$

$$= A + \frac{K_p K_c A - K_p K_c A}{1 + K_p K_c}$$

$$\text{Offset} = \frac{A}{1 + K_p K_c}$$

* K_c should be positive so the offset is small

$$\textcircled{3} \quad G_{\text{cl}} = \frac{\hat{h}_i(s)}{\hat{h}_{\text{isp}}(s)} \quad \hat{h}_i(s) = \hat{Q}_{\text{in}}(s) G_p(s)$$

$$G_a = \frac{\hat{Q}_{\text{in}}(s) G_p(s)}{\hat{h}_{\text{isp}}(s)} \rightarrow \hat{Q}_{\text{in}}(s) = \frac{G_a \cdot \hat{h}_{\text{isp}}(s)}{G_p}$$

$$Q_{\text{in}}(t) = \text{step}(G_a \cdot m) + \bar{Q}_{\text{in}}$$

$$\text{optimum } k_c \text{ at } t=0 \quad h_{\text{isp}} = 52 \text{ cm}$$

$$h_i(0) = \bar{h}_i = 99 \text{ cm}$$

$$Q_{\text{in}}(t) = \bar{Q}_{\text{in}} + k_c(h_{\text{isp}} - h_i(t))$$

$$\frac{110 - 96.166}{(52 - 99)} = k_c$$

$$110 = 96.166 + k_c(8)$$

$$\underline{k_c = 8} \quad \text{max}$$

```

%% Parameters
a=154; %% cm2
C12=10.1; %%cm^(5/2)/s
C23=11; %%cm^(5/2)/s
C3=19.7; %%cm^(5/2)/s

%% Steady State Parameters Part 1
h1s=44; %%cm
Qins=(h1s/(1/(C12^2)+1/(C23^2)+1/(C3^2)))^(1/2)
h2s=Qins^2*(1/(C23^2)+1/(C3^2));
h3s=(Qins/C3)^2;
h1sp=52 %%cm;

%%Constants
k1=C12/(2*sqrt(h1s-h2s));
k2=C23/(2*sqrt(h2s-h3s));
k3=C3/(2*sqrt(h3s));

%%Matrices Part 3
A=[-1*k1/a k1/a 0;k1/a -(k1+k2)/a k2/a;0 k2/a -(k2+k3)/a];
B=[1/a;0;0];
C=[1 0 0];
D=[0];

sys=ss(A,B,C,D);

%%Part 3
G1=tf(sys);

P=pole(G1)

Z=zero(G1)

rlocus(G1)
kc=rlocfind(G1);

%%Tau values
taul=-1/P(1);
tau2=-1/P(2);
tau3=-1/P(3);

%%Theta values
thetal=-1/Z(1);
theta2=-1/Z(2);

%%Process Gain
Kp=dcgain(sys);

%% Closed Loop Transfer Functions -- P only
%%Step Change under P-only Question 5

```

5 & 6

5 & 6

```

h1sp=52; %%cm
M=h1sp-hls %% cm... step size

%%%%%for different kc values
kc1=4;
Gc1=tf(kc1); %% P only controller
kc2=6;
Gc2=tf(kc2); %% P only controller
kc3=8;
Gc3=tf(kc3); %% P only controller
kc4=10;
Gc4=tf(kc4); %% P only controller
Ga=tf(1); %% Assume actuator is very fast
Gs=tf(1); %% Assume sensor is very fast
Gp=G1; %% TF from part 3

Gc=tf(kc); %% P only controller

GcL1=feedback(series(Gc1,Gp),Gs);
GcL2=feedback(series(Gc2,Gp),Gs);
GcL3=feedback(series(Gc3,Gp),Gs);
GcL4=feedback(series(Gc4,Gp),Gs);

h11=step(GcL1*M)+hls;
h12=step(GcL2*M)+hls;
h13=step(GcL3*M)+hls;
h14=step(GcL4*M)+hls;

%Transfer function interms of Q and h1sp
Gq1=tf(GcL1/Gp);
Gq2=tf(GcL2/Gp);
Gq3=tf(GcL3/Gp);
Gq4=tf(GcL4/Gp);

Qin1=step(Gq1*M)+Qins;
Qin2=step(Gq2*M)+Qins;
Qin3=step(Gq3*M)+Qins;
Qin4=step(Gq4*M)+Qins;

%% for chosen kc
GcL=feedback(series(Gc,Gp),Gs);
Gq=tf(GcL/Gp);
Qin=step(Gq*M)+Qins;

%%%%%for different kc values
figure
subplot(2,1,1)
step(GcL1*M+hls,'b',GcL2*M+hls,'r',GcL3*M+hls,'g',GcL4*M+hls,'m')
legend(num2str(kc1),num2str(kc2),num2str(kc3),num2str(kc4))

```

```
xlabel('time')
ylabel('height of tank 1 (cm)')
title('h1 under P-only at various kc values')
subplot(2,1,2)
plot(Qin1,'b')
hold on
plot(Qin2,'r')
hold on
plot(Qin3,'g')
hold on
plot(Qin4,'m')
legend(num2str(kc1),num2str(kc2),num2str(kc3),num2str(kc4))
title('Qin under P-only at several kc values')
axis([0 14 40 150])
xlabel('time (s)')
ylabel('Qin (cm^3/s)')
hold off
hold on

%%%%% for chosen kc value %% Question 6
chosen_kc=kc
offset_of_chosen_kc=(1+Kp*kc)
h1=step(GcL*M)+h1s;
figure
subplot(2,1,1)
plot(h1)
xlabel('time (s)')
ylabel('height of tank 1 (cm)')
title(['h1 under P-only with kc=' num2str(kc) ''])
subplot(2,1,2)
plot(Qin)
xlabel('time (s)')
ylabel('Qin (cm^3/s)')
title([' Required Qin for kc= ' num2str(kc) ''])
hold on
```

Qins =
46.1666

hlsp =
52

P =
-0.0391
-0.0170
-0.0025

Z =
-0.0389
-0.0126

Select a point in the graphics window

selected_point =
-0.0598 + 0.0000i

M =
8

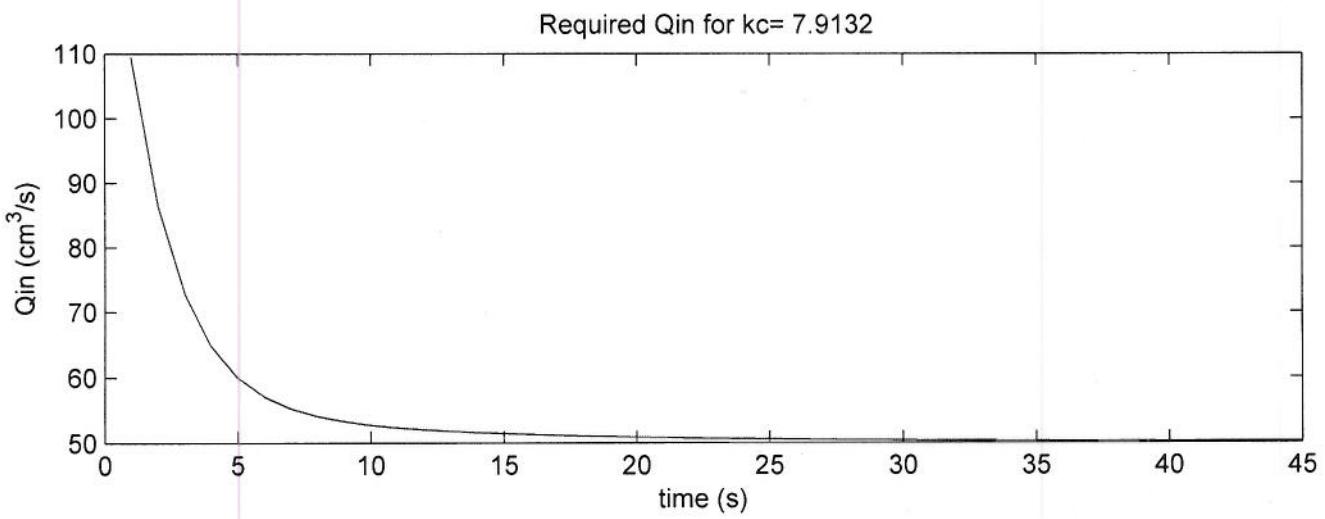
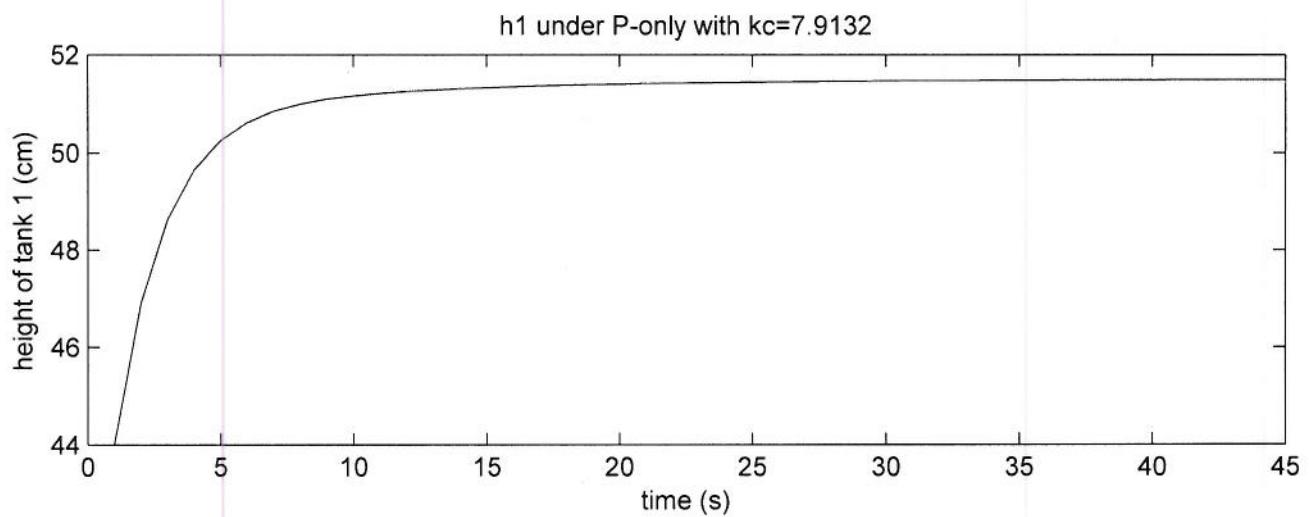
* chosen_kc = \varnothing (6)
7.9132

* offset_of_chosen_kc = \star (6)
16.0837

>>

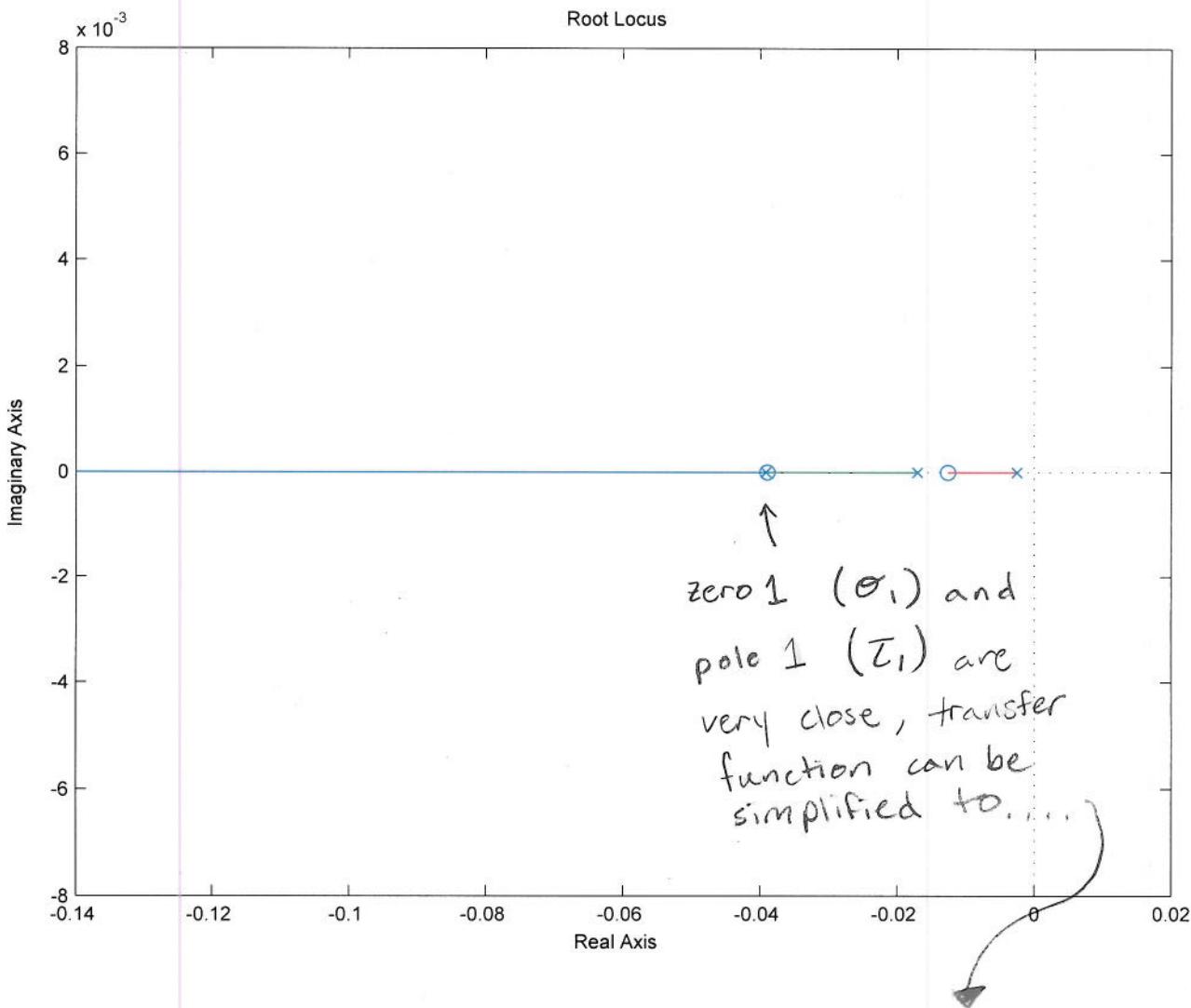
Part 6

Chosen K_C



PART 5)

original: $G_p = \frac{1.906(25.7s+1)(79.6s+1)}{(25.6s+1)(58.7s+1)(400.1s+1)}$



simplified:
trns fxn:

$$G_p(s) = \frac{1.906(79.6s+1)}{(58.7s+1)(400.1s+1)}$$

$$G_p(s) = \frac{k_p (\sigma_2 s + 1)}{(\tau_2 s + 1)(\tau_3 s + 1)}$$

5

Characteristic Equation: $(\tau_2 s + 1)(\tau_3 s + 1) + K_p K_c (\theta_2 s + 1)$

$$= \tau_2 \tau_3 s^2 + \tau_2 s + \tau_3 s + 1 + K_p K_c \theta_2 s + K_p K_c$$

$$= \tau_2 \tau_3 s^2 + (\tau_2 + \tau_3 + K_p K_c \theta_2) s + K_p K_c + 1$$

$$= 23479 s^2 + (458.78 + 151.674 K_c) s + 1.906 K_c + 1$$

Routh array

1	23479	1.906 K _c + 1
2	458.7769 + 151.674 K _c	0
3	$\frac{(458.78 + 151.674 K_c)(1.906 K_c + 1) - 0}{(458.78 + 151.674 K_c)}$	
	$\downarrow 1.906 K_c + 1$	

$89.083 x^2 + 1026.2x + 458.78 > 0$

$29.5402 K_c - 7.98$

$K_c > -3.025$

$K_c > -0.5246$

$1.906 K_c + 1 > 0 \quad 1.906 K_c > -1$

$K_c = -0.525$ for stability

$\hat{h}_1(s) = G_{CL}(s) * \hat{h}_{ISP}(s)$

- ⑥ Graphs show that as K_c increases, the offset decreases. However, the maximum value of K_c to stay within limits of the pump is $K_c = 8$.

7) PI control

Simplified $G_P = \frac{K_p(\Theta_2 s + 1)}{(\tau_2 s + 1)(\tau_3 s + 1)}$

$$= \frac{K_p \Theta_2 s + K_p}{\tau_2 \tau_3 s^2 + (\tau_2 + \tau_3)s + 1}$$

$$\begin{aligned} K_p &= 1.906 \\ \Theta_2 &= 79.5725 \\ \tau_2 &= 58.6838 \text{ pole} \\ \tau_3 &= 400.0931 \end{aligned}$$

$$G_C = K_C \left(1 + \frac{1}{\tau_I s} \right) = \frac{K_C s + \frac{K_C}{\tau_I}}{s}$$

$$G_{extended} = \frac{K_p \left(1 + \frac{1}{\tau_I s} \right) (\Theta_2 s + 1)}{(\tau_2 s + 1)(\tau_3 s + 1)} = \frac{K_p \left(s + \frac{1}{\tau_I} \right) (\Theta_2 s + 1)}{s(\tau_2 s + 1)(\tau_3 s + 1)}$$

$$= \frac{K_p \left(s^2 \Theta_2 + s + \frac{\Theta_2 s}{\tau_I} + \frac{1}{\tau_I} \right)}{s(\tau_2 \tau_3 s^2 + \tau_2 s + \tau_3 s + 1)}$$

$$= \frac{K_p s^2 \Theta_2 + K_p \left(1 + \frac{\Theta_2}{\tau_I} \right) s + \frac{K_p}{\tau_I}}{\tau_2 \tau_3 s^3 + (\tau_2 + \tau_3) s^2 + s}$$

3 poles
2 zeros

Scenarios:

- A $\tau_I > 400$
- B $59 < \tau_I < 400$ *
- C $\tau_I < 59$

From root locus Diagrams

A: no oscillations / slow

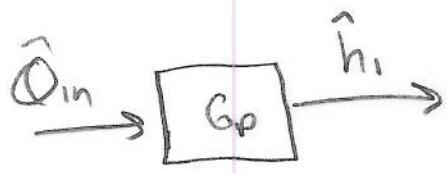
B: some oscillations / fast

C: some oscillations / slow due to pole and zero close together

$$\theta_{in}(t) = \text{step}(G_{\Theta} \cdot m) + \bar{\theta}_{in}$$

$$G_P = \frac{G_u}{G_p}$$

7) Pairs of K_c & τ_I for closed loop system to be stable.

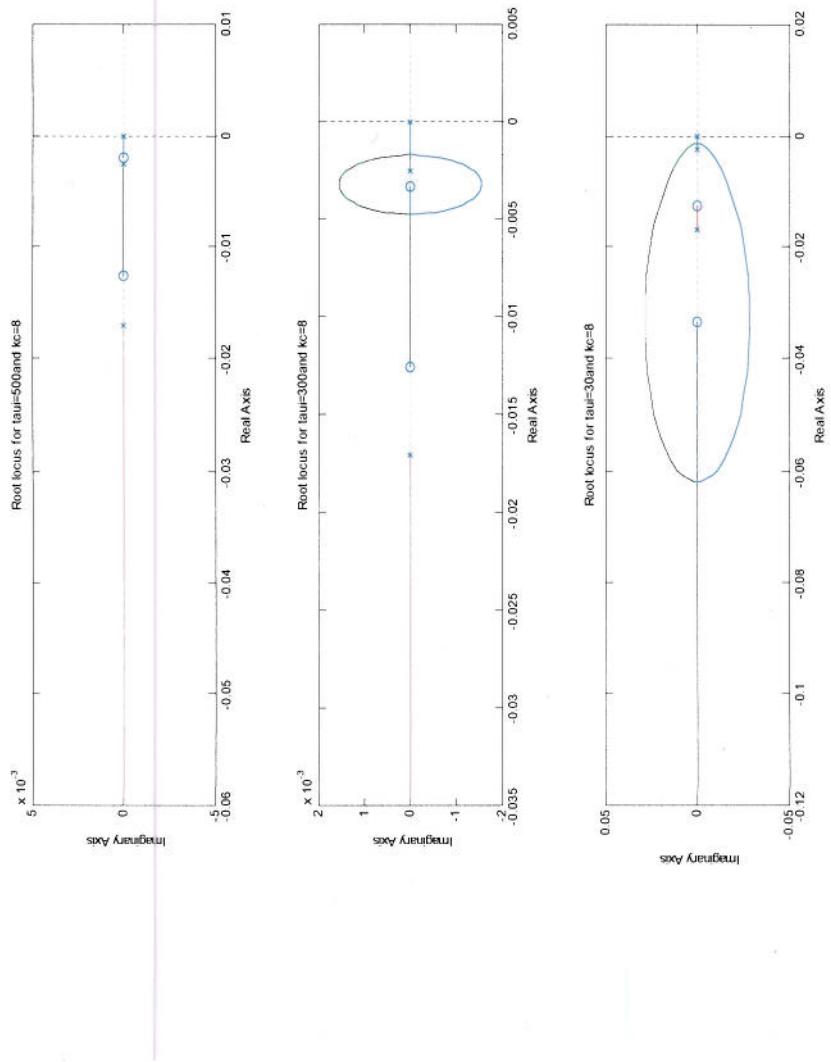


$$G_p = \frac{K_p(\theta_1 s + 1)(\theta_2 s + 1)}{(\tau_1 s + 1)(\tau_2 s + 1)(\tau_3 s + 1)}$$

• in part 3 we showed that the poles have negative real parts
Therefore, G_p is stable.

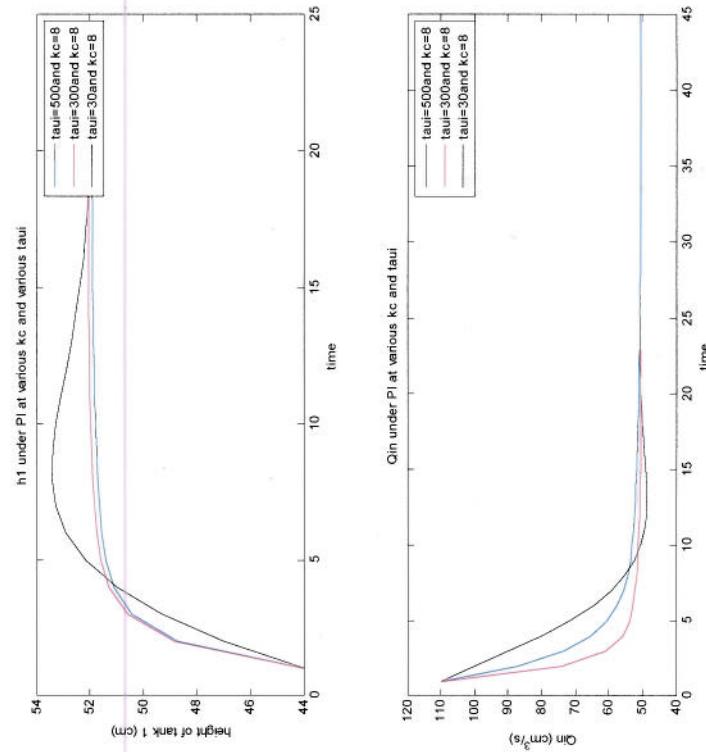
Since \hat{Q}_{in} is bounded by the limits of the pump & G_p is stable, \hat{h}_1 will always be stable for all pairs of K_c & τ_I .

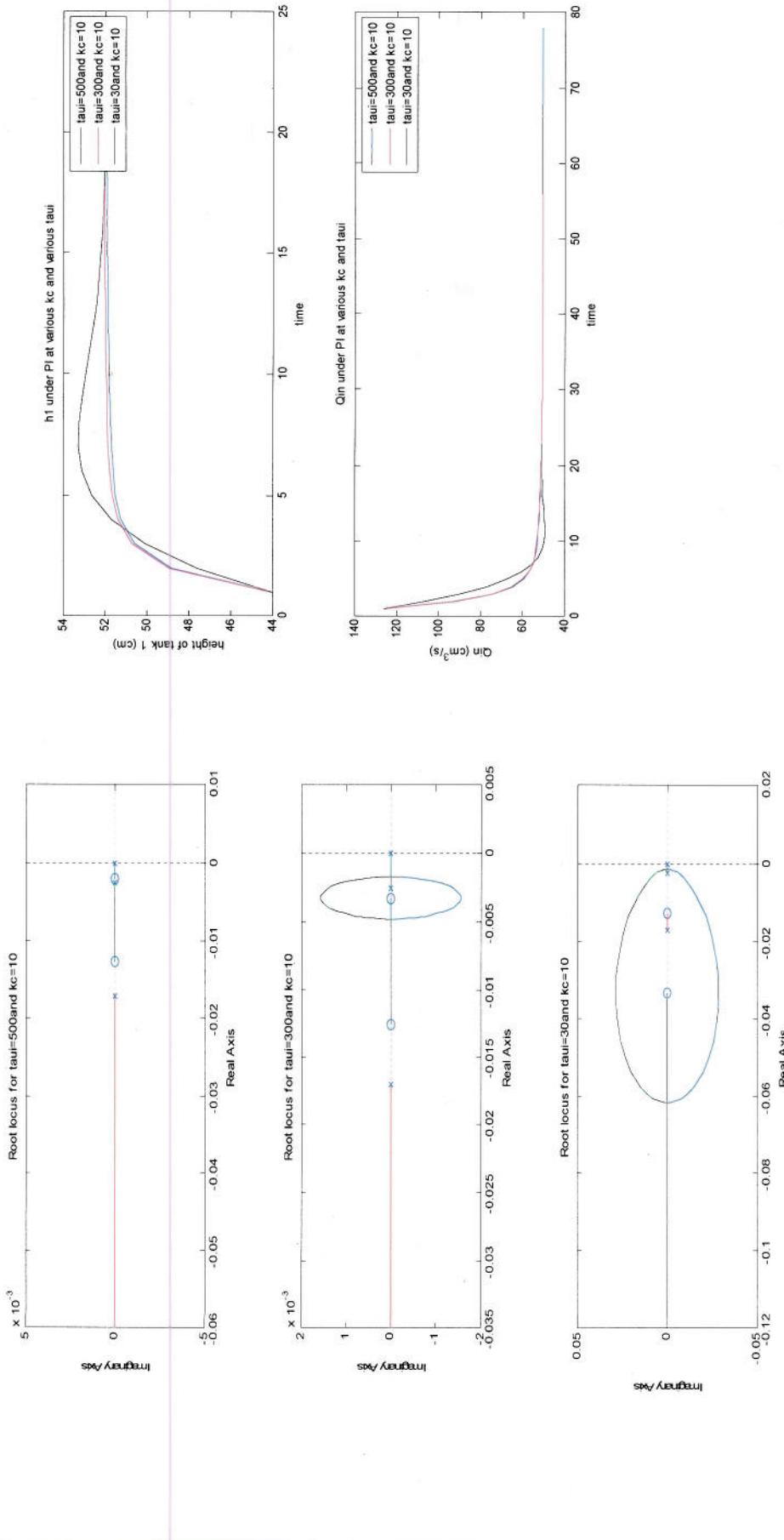
Part 7



$K_c=8$

- Pump limits are respected
- $\tau_i=30$ results in overshoot (response time ~ 20 sec)
- $\tau_i=500$ results in a slower response (response time ~ 18 sec)
- $\tau_i=300$ results in the fastest response (response time ~ 15 sec)

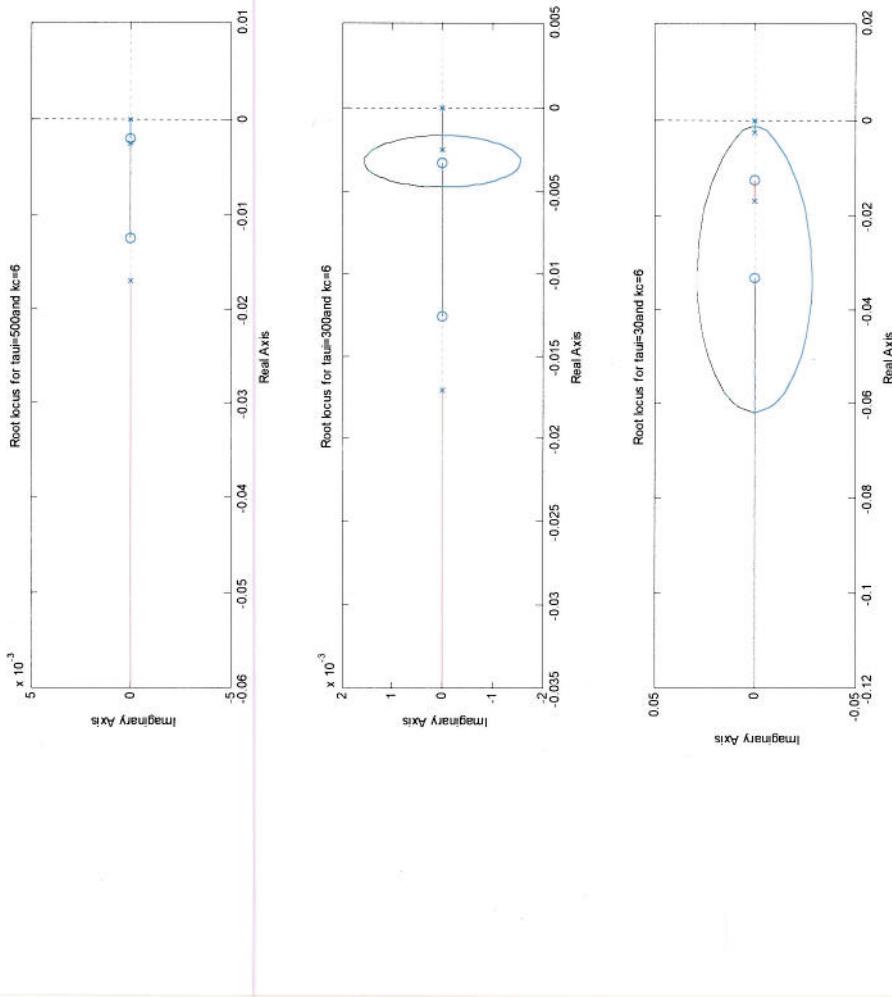




Kc=10

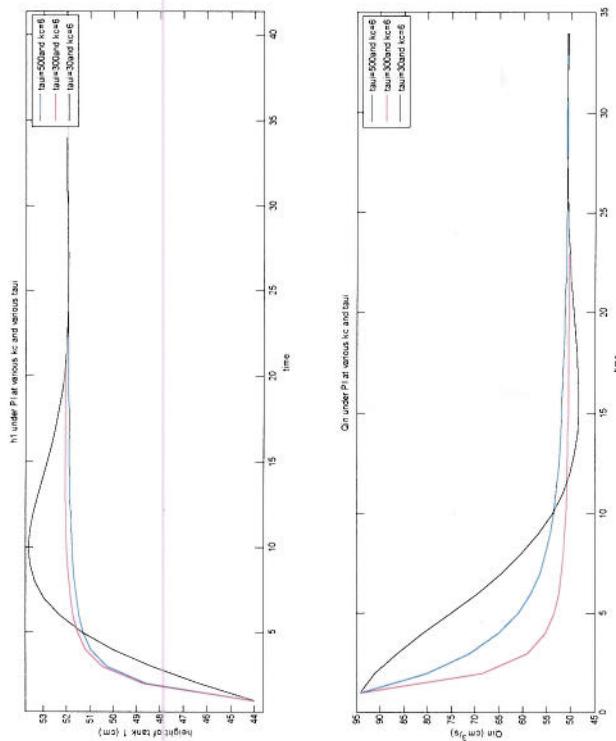
- Pump limits are not respected
- Taui=30 results in overshoot (response time ~ 18 sec)
- Taui=500 results in a slower response (response time ~ 17 sec)
- Taui=300 results in the fastest response (response time ~ 16 sec)

Part 7



$K_c=10$

- Pump limits are respected
- $\tau_i=30$ results in overshoot (response time ~ 24 sec)
- $\tau_i=500$ results in a slower response (response time ~ 23 sec)
- $\tau_i=300$ results in the fastest response (response time ~ 20 sec)

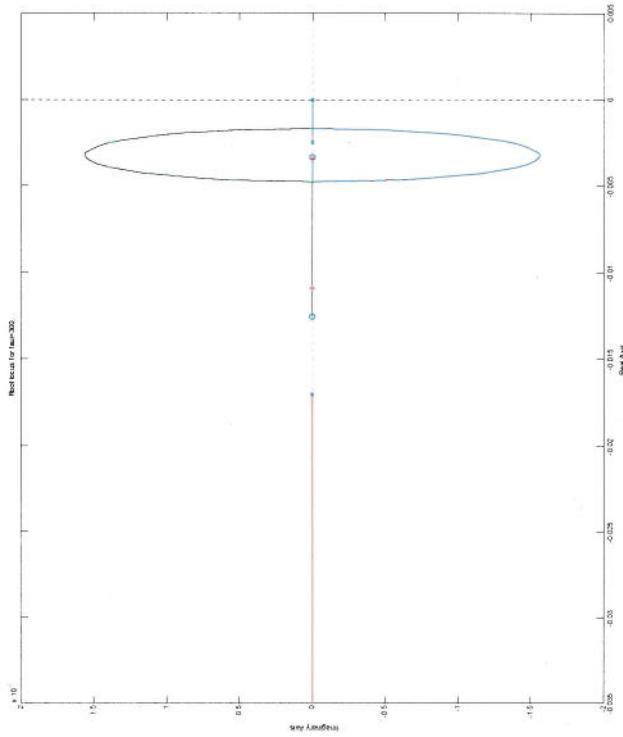
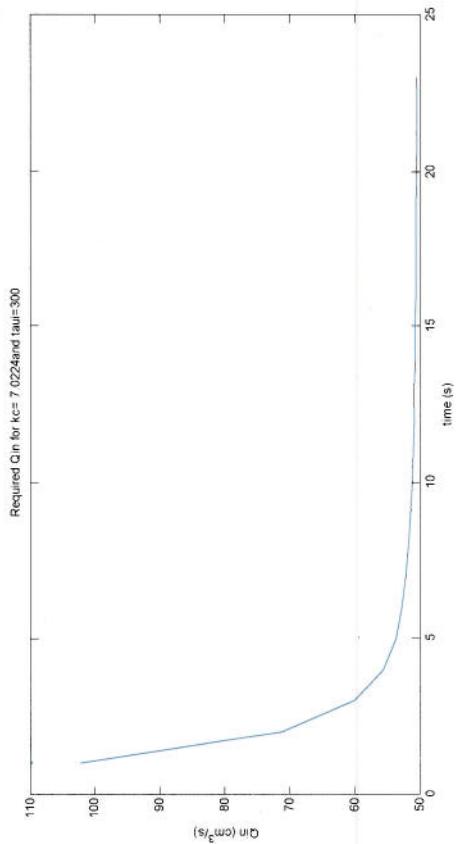
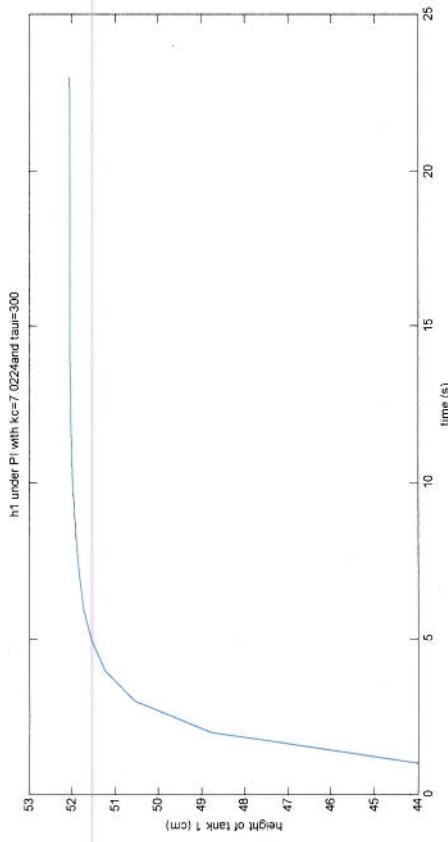


The graphs above show that $\tau_i=30$ is too much integral action \rightarrow overshoot and slower response.

The graphs above show that $\tau_i=500$ is too little integral action \rightarrow slow response.

Therefore, $\tau_i=300$ is the best option of the 3 and will be chosen for part 8. Also, $K_c=8$ results in the best response.

PART 8



- This pair gives a reasonable response (fast (~9sec) with no oscillations and no overshoot)

```
%% Parameters
a=154; %% cm2
C12=10.1; %% cm^(5/2)/s
C23=11; %% cm^(5/2)/s
C3=19.7; %% cm^(5/2)/s

%% Steady State Parameters Part 1
h1s=44; %% cm
Qins=(h1s/(1/(C12^2)+1/(C23^2)+1/(C3^2)))^(1/2)
h2s=Qins^2*(1/(C23^2)+1/(C3^2));
h3s=(Qins/C3)^2;
h1sp=52; %% cm;

%% Constants
k1=C12/(2*sqrt(h1s-h2s));
k2=C23/(2*sqrt(h2s-h3s));
k3=C3/(2*sqrt(h3s));

%% Matrices Part 3
A=[-1*k1/a k1/a 0;k1/a -(k1+k2)/a k2/a;0 k2/a -(k2+k3)/a];
B=[1/a;0;0];
C=[1 0 0];
D=[0];

sys=ss(A,B,C,D);

%% Part 3
G1=tf(sys);

P=pole(G1);

Z=zero(G1);

%% Tau values
tau1=-1/P(1);
tau2=-1/P(2);
tau3=-1/P(3);

%% Theta values
thetal=-1/Z(1);
theta2=-1/Z(2);

%% Process Gain
Kp=dcgain(sys);

%% Closed Loop Transfer Functions -- P only
%% Step Change under P-only Question 5
h1sp=52; %% cm
M=h1sp-h1s; %% cm... step size
```

```
%% prompt kc
prompt='Choose a kc?';
kcp= input(prompt)

%%%% transfer functions in process
Ga=tf(1); %% Assume actuator is very fast
Gs=tf(1); %%Assume sensor is very fast
nums=[Kp*theta2 Kp];
dens=[tau2*tau3 tau2+tau3 1];
Gps=tf(nums,dens); %% Simplified Transfer Function

%%%Extended Transfer Function using simplified Gcl
%%Case A
tauia=500;
numa=[Kp*theta2 Kp*(1+theta2/tauia) Kp/tauia];
dena=[tau2*tau3 (tau2+tau3) 1 0];
Gexta=tf(numa,dena);
numca=[kcp kcp/tauia];
denca=[1 0];
Gca=tf(numca,denca); %% PI controller
Gcla=feedback(series(Gps,Gca),Gs);
hla=step(Gcla*M)+hls;
Gqa=tf(Gcla/Gps);
Qina=step(Gqa*M)+Qins;

%%Case B
tauib=300;
numb=[Kp*theta2 Kp*(1+theta2/tauib) Kp/tauib];
denb=[tau2*tau3 (tau2+tau3) 1 0];
Gextb=tf(numb,denb);
numcb=[kcp kcp/tauib];
dencb=[1 0];
Gcb=tf(numcb,dencb); %% PI controller
Gclb=feedback(series(Gps,Gcb),Gs);
h1b=step(Gclb*M)+hls;
Gqb=tf(Gclb/Gps);
Qinb=step(Gqb*M)+Qins;

%%Case C
tauic=30;
numc=[Kp*theta2 Kp*(1+theta2/tauic) Kp/tauic];
denc=[tau2*tau3 (tau2+tau3) 1 0];
Gextc=tf(numc,denc);
numcc=[kcp kcp/tauic];
dencc=[1 0];
Gcc=tf(numcc,dencc); %% PI controller
Gclc=feedback(series(Gps,Gcc),Gs);
h1c=step(Gclc*M)+hls;
```

```
Gqc=tf(Gclc/Gps);
Qinc=step(Gqc*M)+Qins;
%%Root Locus
figure
subplot(3,1,1)
rlocus(Gexta)
title(['Root locus for taui=' num2str(tauia) 'and kc=' num2str(kcp) ''])
subplot(3,1,2)
rlocus(Gextb)
title(['Root locus for taui=' num2str(tauib) 'and kc=' num2str(kcp) ''])
subplot(3,1,3)
rlocus(Gextc)
title(['Root locus for taui=' num2str(tauic) 'and kc=' num2str(kcp) ''])
hold on

%%%Plotting
figure
subplot(2,1,1)
plot(hla,'b')
hold on
plot(hlb,'m')
hold on
plot(hlc,'k')
hold off
title(['h1 under PI at various kc and various taui'])
legend(['taui=' num2str(tauia) 'and kc=' num2str(kcp) ''],['taui=' num2str(tauib) 'and kc=' num2str(kcp) ''],[ 'taui=' num2str(tauic) 'and kc=' num2str(kcp) '' ])
xlabel('time')
ylabel('height of tank 1 (cm)')
subplot(2,1,2)
plot(Qina,'b')
hold on
plot(Qinb,'m')
hold on
plot(Qinc,'k')
hold off
%axis([0 60 40 120])
title(['Qin under PI at various kc and taui'])
legend(['taui=' num2str(tauia) 'and kc=' num2str(kcp) ''],['taui=' num2str(tauib) 'and kc=' num2str(kcp) ''],[ 'taui=' num2str(tauic) 'and kc=' num2str(kcp) '' ])
xlabel('time')
ylabel('Qin (cm^3/s)')

%% for chosen kc and taui - Part 8
prompt2='Choose a taui?';
taui= input(prompt2)
num=[Kp*theta2 Kp*(1+theta2/taui) Kp/taui];
den=[tau2*tau3 (tau2+tau3) 1 0];
Gext=tf(num,den);
```

```
figure
rlocus(Gext)
kc=rlocfind(Gext)
title(['Root locus for taui=' num2str(tau) ''])
hold on
numc=[kc kc/taui];
denc=[1 0];
Gc=tf(numc,denc); %% PI controller
Gcl=feedback(series(Gps,Gc),Gs);
h1=step(Gcl*M)+h1s;
Gq=tf(Gcl/Gps);
Qin=step(Gq*M)+Qins;

%Plotting - Part 8
figure
subplot(2,1,1)
plot(h1)
xlabel('time (s)')
ylabel('height of tank 1 (cm)')
title(['h1 under PI with kc=' num2str(kc) ' and tau=' num2str(tau) ''])
subplot(2,1,2)
plot(Qin)
hold on
y=110
plot(y,:)
hold off
xlabel('time (s)')
ylabel('Qin (cm^3/s)')
title([' Required Qin for kc= ' num2str(kc) ' and tau=' num2str(tau) '])
hold on
```